Stochastic Process

I Course Description

The main content of this course includes: General theory of stochastic processes (Kolmogorov theorem), Poisson process and renewal theorems; Martingales (super-martingales and sub-martingales, Doob-Meyer decomposition, stopping times, fundamental inequalities, convergence theorems, optional sampling theorem); Discrete-time Markov Chains (weak ergodic theorems, invariant distributions, mean recurrence times, limits of transience probability); Continuous-time Markov Chains (Kolmogorov backward equation, Markov transition and rate kernels); Brownian motion (sample paths, strong Markov property and the reflection principle, computations based on passage times, stopping time); Markov processes (semigroup, the martingale problem, strong Markov property, Feynman-Kac formulas); Introduction to stochastic analysis (stochastic integration, Ito formula, the Girsanov theorem, stochastic differential equations, diffusion processes; Stationary processes and ARMA models.

II Contents

1 Introduction

the concept of stochastic process, examples
existence theorem of Kolmogorov.

2 Poisson process and renewal process.

the background of the Poisson process
the distribution of arrival time and time interval between two events,
inhomogeneous Poisson process and the compound Poisson process;
The general renewal process and its limit theorem.

3 Markov chain

3.1 discrete–time Markov chain
Transition matrix
Chapman–Kolmogorov equation
Recurrence and transience
Positive recurrence and null recurrence
Stationary distribution

3.2 Continuous-time Markov chain
Transition matrix
Infinitesimal generator
Kolmogorov's backward equation and forward equation
Stationary distribution
Birth-and-Death process

4 Martingale

Conditional expectation and filtration
Concepts of martingale, submartingale and supmartingale
Doob-Mayer’s decomposition
Stopping times and Optional Sampling Theorem
Martingale Convergence Theorem

5 Brownian motion

Definition
The Markovian property of BM
The martingale property of BM
sampled quadratic variation
The reflected principle and the distribution of the maximum

6 Stochastic calculus

Mean-square convergence and mean-square calculus
Stochastic integral
Ito’s formula
Girsanov’s Theorem
Stochastic differential equation and solution
Black-Scholes Formula

7 Weakly Stationary Process

Weakly stationary process and auto-covariance function
Spectrum and the spectral density
spectral representation
Mean square ergodic theorem
ARMA